# Full waveform inversion with time-dependent receiver-extension: an efficient inner loop optimization

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## SUMMARY

Receiver-based extension strategy introduces additional degrees of freedom to full waveform inversion, in order to improve the fit between the observed and calculated data at early iterations. This helps circumvent the cycle-skipping phenomenon. The additional degrees of freedom are the receiver positions. In this study, we make this relocalization time-dependent, meaning that the receiver positions vary along time. The resulting mathematical problem is a two nested-loops minimization, where the outer loop is the conventional FWI loop to update the subsurface mechanical parameters, and the inner loop aims at finding the optimal time-dependent virtual receivers positions. This inner loop problem is heavily nonlinear and non-convex. Finding the global minimum is therefore a challenging task, and we use for that a computational intelligence technique, the Particle Swarm Optimization (PSO). PSO is a heuristic optimization method relying on an ensemble of realizations of model parameters called particles. This makes it possible to thoroughly explore the search space with few iterations. By taking a close look on the inner loop problem, we introduce an improvement to our PSO implementation by judiciously choosing the starting inner loop parameters. Finally, we show a numerical case study using a North Sea exploration scale 2D synthetic model.

## INTRODUCTION

Full waveform inversion is a PDE-constrained optimization problem, that aims at improving the fit between observed and synthetic datasets iteratively, using gradient-based optimization approaches. The synthetic data are computed using a wave equation operator, which is nonlinear with respect to the model parameters, causing the misfit function to be non-convex. This gives rise to a main challenge, cycle-skipping (Virieux and Operto, 2009), where the optimization converges to a local minimum matching the wrong phases with each other. This occurs when the time shift between the observed and the calculated data is larger than half the dominant period. Many strategies have been developed to overcome this issue. We focus in this study on a receiver-based extension strategy, first introduced by Métivier and Brossier (2022). This strategy is based on the introduction of additional degrees of freedom, the receivers positions, in order to fit the data when the model estimate is poor at the early FWI iterations. This extended problem is solved using a two nested loops strategy, where the outer loop solves for the model parameters, while the inner loop focuses on the optimal virtual receivers positions using a global optimization scheme. The initial method of Métivier and Brossier (2022) has been extended by Benziane et al. (2023) with the introduction of a time-dependent receiver position, in order to improve the fit for multiple arrivals. Introducing the time-dependence requires adding more degrees of freedom to the inner problem, making the use of global optimization very challenging. A practical solution would therefore be to use stochastic approaches such as simulated annealing (Ingber, 1993), as shown in Benziane et al. (2023). However, the inner problem is heavily nonlinear, and finding a solution remains a challenging task. In this study we investigate an alternative heuristic optimization approach, Particle Swarm Optimizer (PSO), for the inner loop problem. We take a close look on the inner loop misfit and we test the method on a 2D North Sea exploration scale model. We show how this strategy improves resilience to cycle-skipping, while being directly applicable to time domain-FWI with manageable extra cost.

## THEORY

The receiver extension strategy introduces additional degrees of freedom to the FWI problem, at the receiver position, allowing the receiver to move as a function of acquisition time. This helps eliminate the kinematic mismatch between the observed and the calculated data. The receiver extension problem is solved using a two nested loops strategy, where the inner loop finds the optimal virtual receiver positions (a time-dependent position). The outer loop is the conventional FWI loop, which updates the model parameters based on the amplitude discrepancy. We write the receiver extension problem as

$$\begin{aligned}\min_{m,\Delta x} f(m,\Delta x) = \frac{1}{2}\sum_{r=1}^{N_r} ||d_{cal,r}[m,\Delta x_r] - d_{obs,r}||^2_{\mathcal{D}} \\ + \frac{\alpha}{2}\sum_{r=1}^{N_r} \frac{||d_{obs,r}||_2^2}{L^2}||\Delta x_r||_2^2 \\ + \frac{\beta}{2}\sum_{r=1}^{N_r} \frac{||d_{obs,r}||_2^2}{V_{max}^2}||\Delta \dot{x}_r||_2^2.\end{aligned} \quad (1)$$

We aim at minimizing the bivariate misfit function (equation 1), where $m$ are the model mechanical parameters, $\Delta x_r(t)$ are the receiver position corrections, while $r$ is the receiver index. The first term in equation 1 is the data fit term, which is the $L_2$ norm of the data residuals in the data space $\mathcal{D}$. The calculated data are obtained by

$$\begin{cases} d_{cal,r}[m,\overline{\Delta x}](x,t) = u[m](x_r + \overline{\Delta x_r}(t),t) \\ A(m)u(x,t) = b(x,t) \end{cases}, \quad (2)$$

where $\overline{\Delta x}(t)$ are the optimal receiver position corrections. The calculated data are extracted at the virtual receivers positions $(x_r + \overline{\Delta x_r}(t))$ from the wavefield $u$, which is computed using the wave equation operator $A(m)$ in equation 2, with $b(x,t)$ being the source time function. The second term in the right hand side of equation 1 penalizes the receiver position correction, in order to prevent it from being too large and to force it to become small along iterations. $L$ is the maximum allowed receiver relocalization distance, and $\alpha$ is a tuning parameter, for weighting this penalty term. Similarly, the third term in the right hand side of equation 1 penalizes the receiver speed (the first order derivative with respect to time of the receiver relocalization $\Delta\dot{x}(t)$). $V_{max}$ is the maximum allowed receiver speed, and $\beta$ is a tuning parameter.

In the following, the receiver extension problem is presented for one source for clarity, the extension to multiple sources is obtained by summation over the sources (receiver relocalization values are computed for each source separately).

In order to obtain the time-dependent receiver position, we use a piecewise polynomial parametrization, given by

$$\Delta x_{r,k}(t) = \sum_{i=1}^{N_\ell+1} a_i \ell_i^{N_\ell}(t), \tag{3}$$

where $\ell^{N_\ell}(t)$ are Lagrange basis functions of order $N_\ell$. The time vector is divided into segments, each of which contains one Lagrange polynomial. Equation 3 shows the receiver position correction for a segment $k$ as a function of the acquisition time. The inner loop aims at finding the optimal $a_i$ values, that define the time-dependent virtual receivers positions. In Benziane et al. (2023), we have shown that it can be beneficial to make the virtual receiver move in two dimensions in the physical space for 2D FWI problems, while keeping the inner loop problem as 1D problem. We do this by allowing the virtual receiver to move along a predefined $(x,z)$ trajectory.

## INNER LOOP OPTIMIZATION

The inner loop problem consists in obtaining a time dependent extended receivers positions, using a polynomial parametrization. At each FWI iteration, a portion of the incident wavefield (decimated in time) is kept in memory, in order to be able to extract synthetic data at the extended receiver positions at every time step, without any recomputation. We illustrate this extraction at the time-dependent receiver position in Figure 1: the calculated data (green line plot) are extracted from the calculated incident wavefield (shown in gray-scale in Figure 1b) at the time-dependent virtual receiver position (white line plot), which is parametrized with two segments and first order Lagrange polynomials, the white circles indicate the values of the relocalization at the control points ($a_i$ in equation 3). Please note that the time-dependent receiver positions are not confined to the finite-difference grid points, but can be extracted at arbitrary locations thanks to Kaiser-windowed sinc interpolation (Hicks, 2002), as the extended receiver position may fall in between grid points.

The inner loop problem consists therefore in obtaining the optimal receivers positions corrections at the control points $a_i$. However, it appears not tractable to solve the inner problem using grid-search optimization, as the size of this inverse problem is too large with the time-dependency. In our previous study, we have investigated the Markov Chain Monte Carlo method Aster et al. (2013), which requires a large number of iterations in order to converge. We have also investigated simulated-annealing methods (Ingber, 1993), which proved to be challenging to tune.

We aim at finding a practical and efficient solution for this inner loop problem. We borrow a technique from computational intelligence: Particle Swarm Optimization, which seems to be a viable candidate. PSO was proposed by Kennedy and Eberhart (1995), it is a heuristic optimization method, where the search space is explored by so-called particles. A swarm contains $N_p$ particles, and each particle explores the search space by its position $\mathbf{x}_j$.

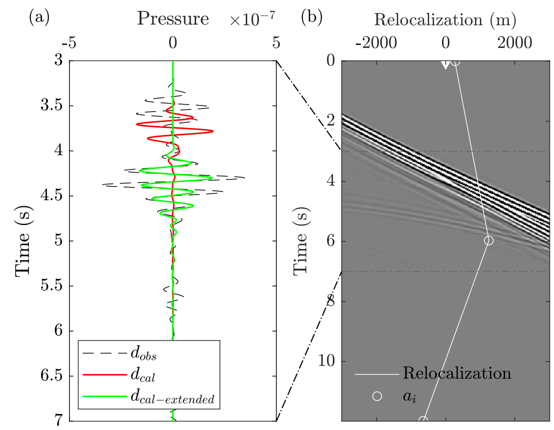


Figure 1: Receiver-extension illustration, (a) data fit: the observed data is shown in a dashed black line, the calculated data in red and the extended calculated data in green. (b) the calculated incident wavefield is shown in gray-scale, for different relocalization values around the original receiver position (white inverted triangle). The white line shows the time-dependent receiver relocalization where the calculated data are extracted, the white circles are the values at the control points.

The best model from the swarm (from all the particles), associated with the lowest cost function value, is referred to as the global best ($x_g$ in the equations below). The personal best ($x_{p,j}$ in the equations hereafter) on the other hand, is the best solution obtained for each individual particle.

Let $\mathbf{x}_j^i$ denotes a particle $j$ position in a search space $\mathbb{R}^N$ at iteration $i$. The particle position is then updated to iteration $i+1$ as such

$$\mathbf{x}_j^{i+1} = \mathbf{x}_j^i + \mathbf{u}_j^{i+1}, \text{ with } \mathbf{x}_j^0 = U(\mathbf{x}_{min}, \mathbf{x}_{max}), \tag{4}$$

where $\mathbf{u}_j^i$, is the particle position update at iteration $i$. The starting particle positions are harvested from a uniform distribution ($U$ in equation 4, where $\mathbf{x}_{min}$ and $\mathbf{x}_{max}$ are the search space bounds). In the literature, $\mathbf{u}^i$, is referred to as the particle speed, and is computed as

$$\mathbf{u}_j^{i+1} = \omega \mathbf{u}_j^i + c_1 \mathbf{r}_1^i : [\mathbf{x}_{p,j}^i - \mathbf{x}_j^i] + c_2 \mathbf{r}_2^i : [\mathbf{x}_g^i - \mathbf{x}_j^i]. \tag{5}$$

The first term is called the inertia term controlling the contribution from the past iteration, with $\omega$ being the inertia weight (typically $\omega \in [0.9, 1.2]$). The second term is the contribution of the best position for each particle, where $\mathbf{x}_{p,j}^i$ is the best personal position for a particle $j$ along its past trajectory. The third term is the contribution of the global best position of the swarm, where $\mathbf{x}_g^i$ is the global best position.

Terms $\mathbf{r}_1^i$ and $\mathbf{r}_2^i$ are random variables vectors of the same dimension as the particle position $\mathbf{x}_j^i$, which are harvested from a uniform distribution, $c_1$ and $c_2$ are constants, usually set to equal values, and $\mathbf{a} : \mathbf{b}$ denotes the term to term product of vectors $\mathbf{a}$ and $\mathbf{b}$. If we wish to give more weight to either component, the constants $c_1$ and $c_2$ may be adjusted accordingly (in this study we set $c_1 = c_2 = 2$). This is the basic form of global best PSO (Engelbrecht, 2007) that we use for this study.

Shi and Eberhart (1998) suggest that decreasing linearly the inertia weight helps improve the PSO performance. This is something we have verified in our application. We manage the parameter bounds with "damped reflection" boundary conditions (Xu and Rahmat-Samii, 2007). In other words, if a parti-

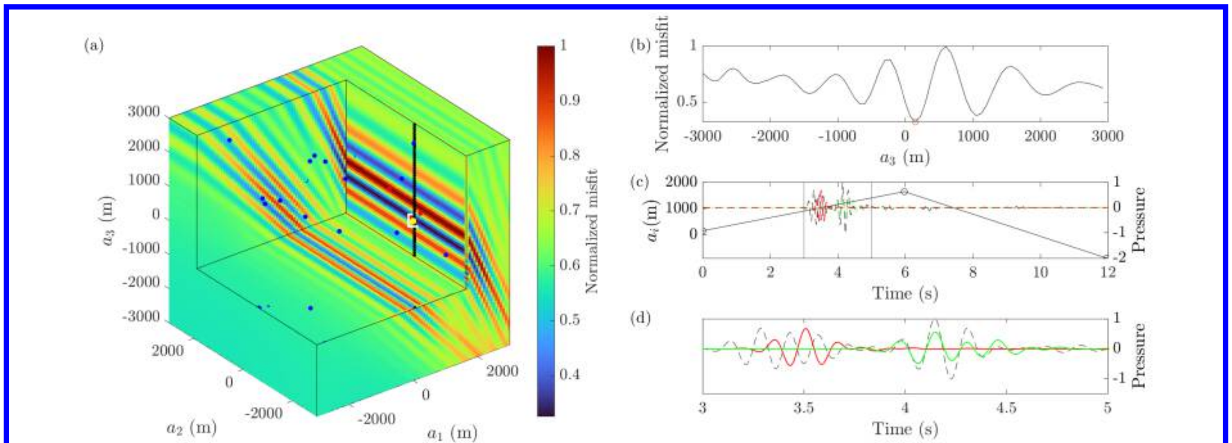


Figure 2: Inner loop problem example for one receiver with 3 degrees of freedom. (a): normalized misfit function, the yellow point is the minimum, the white square indicates the PSO global best, the red dots show each particle best positions at the final iteration. The blue dots are the starting positions for few particles. (b) misfit variation following the $a_3$ axis, shown in a black solid line in (a). (c) relocalization time profile, the circles show the control points $a_i$, each $a_i$ is a dimension in the search space, the observed data are shown in dashed line, the synthetic data are shown in red line and the synthetic data after relocalization are shown in green. (d) shows a zoom of traces shown in (c) between the two vertical lines.

cle wanders beyond the space boundary in a given dimension, it will reflect back inside the domain in the same dimension, however, its position is damped (scaled by $a \leq 1$, where $a$ is a randomly generated number). This is beneficial in our case, as the minimum may be near the domain boundary. The update values (equation 5) can also be bounded, in order to help prevent the particles positions from increasing too fast. We set the maximum allowed update to half the range of a particle position ($\frac{x_{max}-x_{min}}{2}$).

The inner problem misfit function exhibits numerous local minima, therefore generating a starting swarm pseudo-randomly might not be optimal. Having the particles distributed evenly in the search space, can enhance their searching ability (Richards and Ventura, 2004). A simple way of achieving this, is by using an orthogonal particles positions initialization (Yiu-Wing Leung and Yuping Wang, 2001).

We demonstrate the benefit of orthogonal initialization in Figure 3. The global best is shown as a function of iterations for orthogonal and random initializations (the black horizontal lines show the solution obtained with brute-force search for the purpose of this example). Convergence towards a good solution is obtained with fewer iterations with orthogonal initialization as opposed to random initialization. In our algorithm, we use this strategy for the first receiver at the first FWI iteration. Then, for the remaining receivers and subsequent FWI iterations, we take benefit of the previous optimized receiver (or the previous FWI iteration) to initialize the swarm, in way that the particles are scattered around this optimal solution.

## NUMERICAL TEST ON A NORTH SEA EXPLORATION SCALE SYNTHETIC MODEL

In order to illustrate PSO for our case, we use a North Sea exploration scale 2D synthetic model (Figure 5). We parametrize our time-dependency with two segments with first order Lagrange polynomials (equation 3), leading to three degrees of freedom per seismic trace in the inner loop problem. In this example, we use only horizontal relocalization in order to keep the virtual receiver in the same medium with the same properties (the water layer). We present in Figure 2a the inner loop misfit function, with the starting position of few particles in blue.

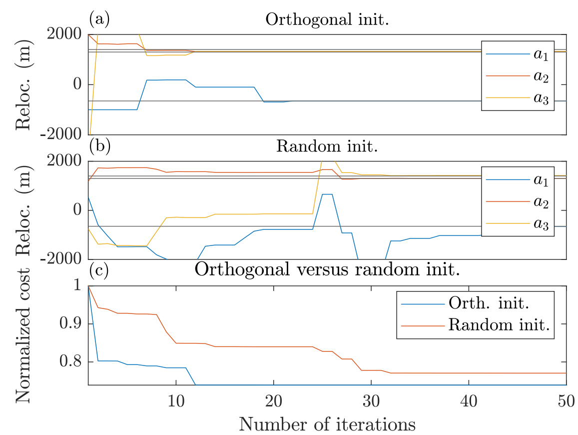


Figure 3: Global best as a function of the number of iterations using (a) orthogonal initialization, and (b) random initialization. (c) shows the cost value associated with the global best, at each iteration for both cases.

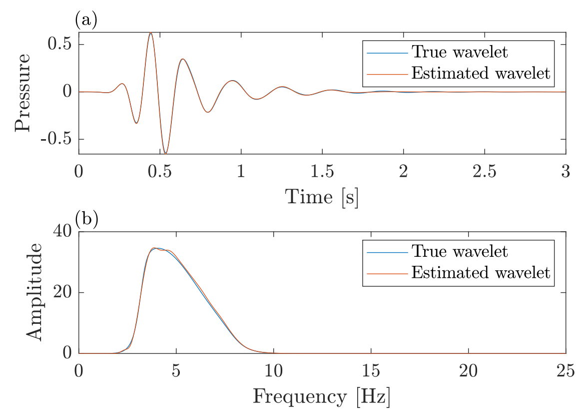


Figure 4: Source wavelet inversion, (a): time-domain, (b) amplitude spectrum.


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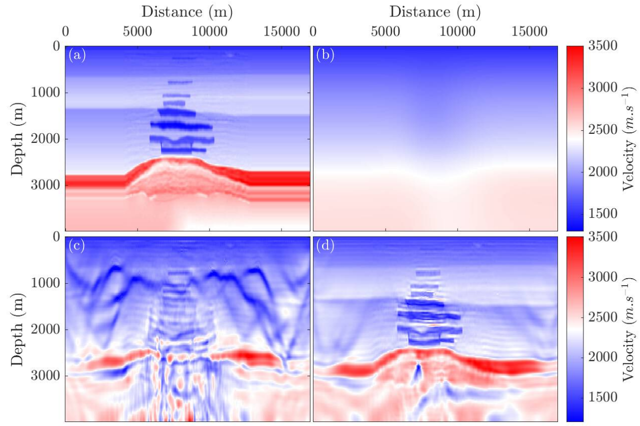


Figure 5: A North Sea exploration scale 2D synthetic model, (a): true model, (b): starting model, (c): final model obtained with conventional FWI and (d): final model obtained with receiver extension strategy.

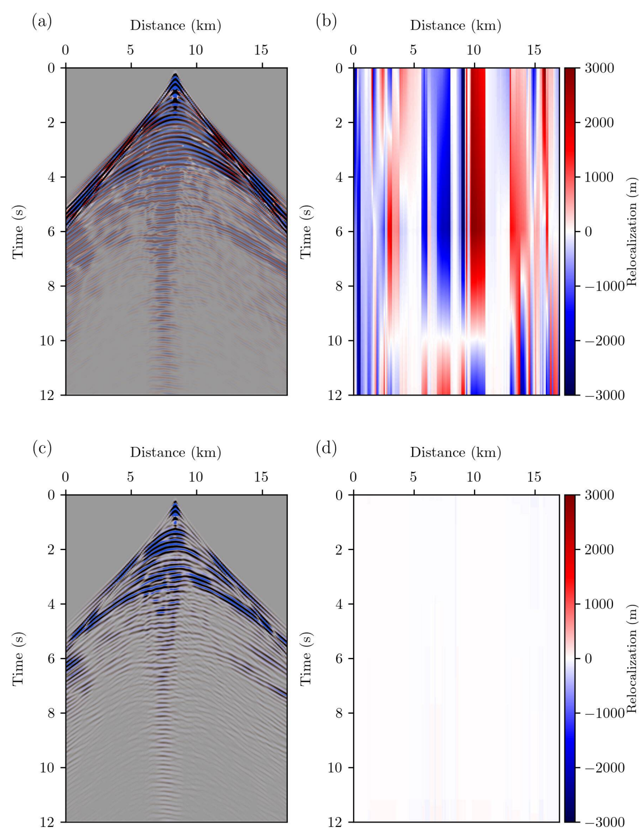


Figure 6: Data fit (a,c) and relocalization gather (b,d), for the first (a,b) last (c,d) FWI iterations. The calculated data are shown red and blue color map, and the observed is shown in gray scale, if the observed and calculated data are in phase, only blue and black are visible, otherwise, red can be seen.

Those starting positions of the swarm allow each particle to explore different regions of the search space. The global minimum is indicated in yellow, it coincides with the PSO global best shown as a white square. The personal best positions for few particles at the last iteration are presented as red dots. We show the misfit variation following the $a_3$ axis in Figure 2b.

The relocalization profile corresponding to the global solution, is shown in Figure 2c, to which we superimpose the observed and synthetic traces, a zoom on the traces is shown as well in Figure 2d. Looking at the misfit plots, we can clearly see the difficulty of the problem, finding the global minimum not being a trivial task.

We carry out a FWI test under the acoustic approximation. We use a synthetic North Sea exploration scale 2D model (Figure 5a) to create the observed dataset (a variable density from this model is used as well, although it is not shown). The starting velocity model is obtained using strong Gaussian smoothing of the true model, with a correlation length of 2000 m (Figure 5b). The starting density is obtained using Gardner's law from the starting velocity model, given by $\rho(x) = 1740(10^{-3}V(x))^{\frac{1}{4}}$.

During the inversion, only the velocity is updated. First, a source wavelet inversion is performed (Pratt, 1999) (Figure 4). For our extended-problem FWI we set the tuning parameter $\alpha$ to 0.001 and cancel out the receiver speed constraint as it is not required for this "small" problem ($\beta = 0$). We run the outer inversion using a preconditioned l-BFGS (Nocedal, 1980).

The data fit and extended receiver position are shown in Figure 6 for one early and last iteration, showing how the relocalized positions converge toward small value along the nested loop optimization. Looking at the final model (Figure 5d), the low velocity anomaly in the center of the model is fully reconstructed, the top portion of the high velocity basement is retrieved as well, however imaging below it remains challenging. A conventional FWI result is shown for the purpose of comparison (Figure 5c). As expected, it suffers from cycle skipping and it is unable to reconstruct the velocity model.

## CONCLUSIONS AND PERSPECTIVES

In this study we tackle the inner loop problem of receiver-extension FWI, which proved to be challenging, using a computational intelligence technique (PSO). This optimizer allows the search space to be explored thoroughly with a manageable cost.

We have tested our method on a North Sea exploration scale synthetic 2D model, obtaining promising results. Our PSO algorithm convergence has been improved thanks to an efficient orthogonal swarm initialization.

Extending the method to 3D FWI as well an application to real data are the next step of this project. This extension should take benefit of the same technique for receiver relocalization, by allowing the virtual receiver to move in a plane defined by the source, the receiver and depth axis along a predefined trajectory to keep this inner problem in 1D. The computational complexity of the inner problem would therefore remain the same in the 3D case for one trace, allowing to give an efficient extended scheme in 3D.

## ACKNOWLEDGEMENTS

This study was partially funded by the SEISCOPE consortium (`http://seiscope2.osug.fr`), sponsored by AKERBP, CGG, DUG, EXXONMOBIL, GEOLINKS, JGI, PETROBRAS, SHELL, SINOPEC and TOTALENERGIES. This study was granted access to the HPC resources provided by the GRICAD infrastructure (https://gricad.univ-grenoble-alpes.fr), Cray Marketing Partner Network (`https://partners.cray.com`) and CINES/IDRIS/TGCC under the allocation 046091 made by GENCI.

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